Mistakes and Other Classroom Techniques

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Introduction

As teachers, we wish to do more than present to our students the established ideas and facts of our fields. We want to give them a sense of how one thinks and creates within the discipline, to impart the tools of scholarship. In my field, mathematics, this concern has led to an increased focus on the teaching and learning of problem solving. Researchers have directed much attention to the executive functions and metacognition involved in problem solving, that is, the solver's awareness of thinking processes and of progress toward a solution (Schoenfeld, 1985). In addition to these procedural matters, attitudes and emotions surrounding mathematical problem solving also have been addressed in the active field of mathematics education research (Silver, 1985). In fact, the concern with thinking patterns and scholarship extends well beyond the technical sphere. Can we incorporate the teaching of these thinking skills into our regular classes? A large body of research indicates that we can, with a few simple techniques.

The Theory

Bandura's social learning theory emphasizes learning through observation (1977, 1986). In most cases, the observer performs the learned behavior embellished with his or her own idiosyncrasies,
rather than imitating precisely the model's actions. This indicates that the observer has processed and integrated what was modeled. Such flexibility in learning is well suited to the teaching of intellectual skills such as mathematical problem solving and critical thinking, because the student must adapt the learned behavior for use in similar, but not identical situations. Problem solving calls for improvisation. When attempting to apply ill-defined rules, trial and error and the ability to find and cope with error become extremely important.

**Mistakes**

Many instructors know the uncomfortable feeling that occurs when a student finds an error in the presentation or a difficulty arises while demonstrating a problem. The legendary professor, upon discovering a mistake in the lecture, says to the students, "I just made that mistake to see if you were all on your toes!" It is an old, but revealing, story. The embarrassed instructor does not want to admit that mistakes (indeed dumb mistakes) can be made even by the expert. Yet in mistakes lurk valuable opportunities to improve student understanding. By openly displaying our reactions to a mistake, we can demonstrate not only the problem-solving process, but also the coping mechanisms for dealing with the emotions that accompany mistakes.

Most instructors present themselves as masters of a subject when they lecture, offering only one neat, final path to the answer and seldom indicating that several correct solutions may be possible. When the correct solution or conclusion is discussed, it usually is presented as a series of steps, each justified by the application of some rule or fact. The reasoning behind the method of arriving at the answer or conclusion rarely is discussed. An alternative is to present a "coping model" to demonstrate that mistakes are a normal part of intellectual life and that it is important to develop the ability to suspect, detect, and correct errors. Even minor mistakes should be pointed out, because they can lead to incorrect results. By sharing small tricks for preventing or finding mistakes, we can help students who may not have developed these skills or even thought to develop them.
Over the years, I have made the usual quota of mistakes in the classroom, either in working a problem that I have not had a chance to prepare or in following student suggestions for solutions. I have learned to take advantage of mistakes to discuss subtle points of mathematical reasoning that I would not have included in a planned lecture. By going through the process of solving the problem rather than just presenting a prepared solution, I model problem-solving skills that could not be explicated effectively in a lecture. Of course, one might ask how mistakes affect the instructor's credibility. Do students lose confidence in a teacher who stumbles occasionally when solving a problem? No instructor wishes to appear ignorant or unprepared, and both common sense and research indicate that as a model's status decreases, so does learning from that model. The message that mistakes are a fact of mathematical life would be lost if students did not perceive the instructor as credible. Fortunately, my experience, as documented in student evaluations of demonstration of knowledge of the subject and preparation for each class meeting, indicates that even when mistakes are highlighted and discussed in class, the students' perception of the instructor's expertise is not damaged.

Coping With Emotions

When I deal with a mistake in the classroom, I model far more than mathematical skills and techniques. As I work through a problem, students see an expert in the field not only solving a problem, but also, like them, feeling puzzled and frustrated, yet working through to a solution. This demonstration of emotion is an important classroom tool. Any mathematics problem worth doing does not present its solution readily. Like an artist facing a blank canvas, the mathematician needs courage to make a start. The first approach often is incorrect, so one must not only find and correct the mistake, but also handle the accompanying disappointment and frustration. For students who have seen the instructor perform only flawlessly, the real experience of problem solving can make them feel incompetent and defeated. Therefore, it is important to help students develop the ability to cope with the anxiety that surfaces in the learning process.

Accordingly, in class I act out my responses to the emotions that
arise from my work. I point out the things I dislike doing (such as long, tedious calculations) and share the joy of discovering an ingenious, smooth argument. I express disappointment when I make a mistake and frustration when I cannot solve a problem immediately. This shows that the correct solution or argument may not always be right at hand and that patience is needed to unravel a difficult problem. As students see an expert displaying the negative emotions they feel and yet eventually achieving success, they are encouraged to persist in their own work.

**Self-Talk**

Demonstrating self-talk is at the heart of my classroom techniques. The classroom mistake presents the opportunity for me to talk through a problem, and in some cases, to verbalize emotions and the accompanying coping responses. I teach students to talk to themselves as they work on problems. Self-talk can be useful in considering mathematical concepts (e.g., "I shouldn't just read the word 'logarithm,' but remind myself what that means."), regulating oneself during calculations (e.g., "Take each step in turn, and don't rush. I'll finish this correctly if I'm careful."), and developing positive attitudes toward problem solving (e.g., "Okay, so I don't know what to do right away; I don't have to give up. I'll start by rereading the problem and making sure I know what each word and phrase means.").

**Conclusion**

The techniques described are only the beginning of changes in our teaching that can be derived from social learning theory. As we become more familiar with these ideas, new ways of teaching will develop naturally, and the skilled actions of the good teacher will be better understood. Our mistakes, when shared, illuminate much of the real, everyday experience of study and scholarship, and give students the tools and the courage to apply their own talents.

**References**

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